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RESEARCH MEMORANDUM

PRINCIPLES OF THE REAC

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PRINCIPLES OF THE REAC

A. S. Mengel

Introduction

The purpose of this report is to acquaint potential customers with the capabilities of the Reeves Electronic Analogue Computer (REAC) now in operation at RAND.

The REAC does with electric circuits what the mechanical differential analyzer does with gears, shafts, and discs, but operates roughly at ten times the speed and one-tenth the accuracy. A problem of average complexity requires a few hours of REAC preparation and testing, after which individual runs can be made in a few minutes. The components of the machine have been adjusted to make the error of a single operation (addition, integration, multiplication, etc.) less than .1 percent of full scale.

The machine will be particularly useful in the solution of nonlinear or implicit equations, or systems of equations which are extremely difficult to handle analytically or numerically.

In an electronic differential analyzer, quantities are represented by voltages. The constant of proportionality between the voltage at a point in the circuit and the quantity it represents is referred to as the scale factor. It is the number of volts representing one unit of the quantity. In the REAC, variables must lie between ± 100 volts, since greater values produce erroneous results. Lights and bells indicate any variable exceeding these limits. The next section will present the components which permit summation, integration, multiplication, etc., of these variables; the principles of the components are given in the appendix.

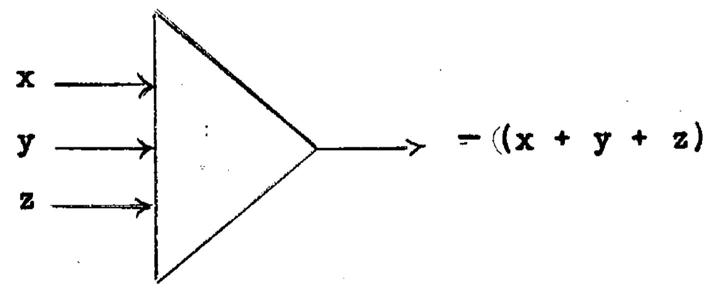
Components

The basic components of the REAC are 23 amplifiers that differ only in feedback elements, relay arrangements, the

number of inputs, and the gains of the inputs. The amplifiers can be conveniently interconnected by patch cords on a panel resembling a telephone switchboard.

All 23 amplifiers can be used to give the negative of the sum of the input voltages and when used as summing amplifiers are drawn in schematic form as follows:

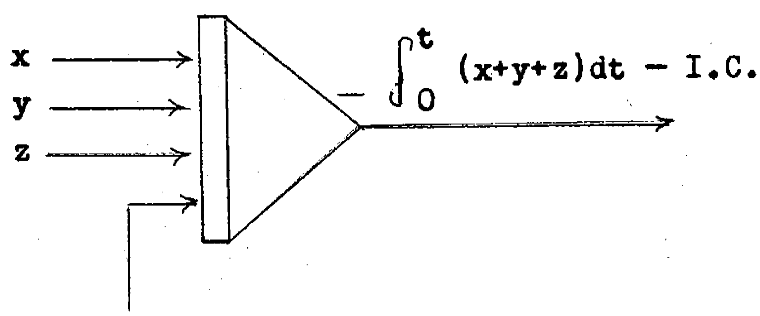
SUMMING AMPLIFIER



Gains other than unity (10, 4, 1/4, 1/10) are available on some amplifiers and seven of the amplifiers can sum as many as seven variables.

Sixteen of the amplifiers can be connected to give the integral (with respect to time) of the sum of the inputs plus an initial condition, with a change of sign as in the summing amplifiers. Integrators are drawn as follows:

INTEGRATOR



I.C. = initial condition

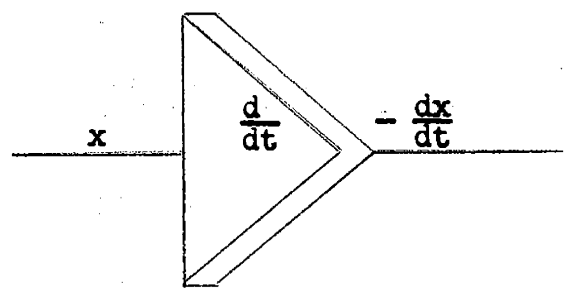
The initial condition values are set by dials located above the amplifier panel.

Differentiation is possible with three of the amplifiers, and this feature is usually employed to permit integration with respect to a variable other than the independent variable, time, as illustrated in the following equation:

$$\int f(x)dx = \int f(x) \cdot \frac{dx}{dt} dt$$

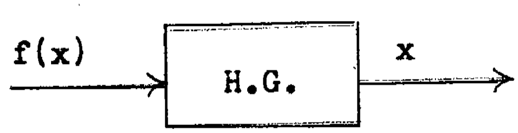
A differentiator is shown as follows:

DIFFERENTIATOR



A RAND modification of the REAC permits use of 16 of the amplifiers as high-gain amplifiers, which are extremely useful in handling implicit or transcendental equations and often ease the planning of circuits, especially when instability is possible. As an example of the use of such an amplifier, consider solving $f(x) = 0$. The output of the high-gain amplifier is specified as x. The variable x is operated upon to yield f(x), which is fed back into the input of the high-gain amplifier. The feedback nature of the circuit will drive f(x) towards zero, yielding the value of x satisfying $f(x) = \varepsilon$, where ε equals x divided by the gain of the amplifier (approximately 30,000) and hence is zero within the accuracy of the machine. A high-gain amplifier is indicated as shown below:

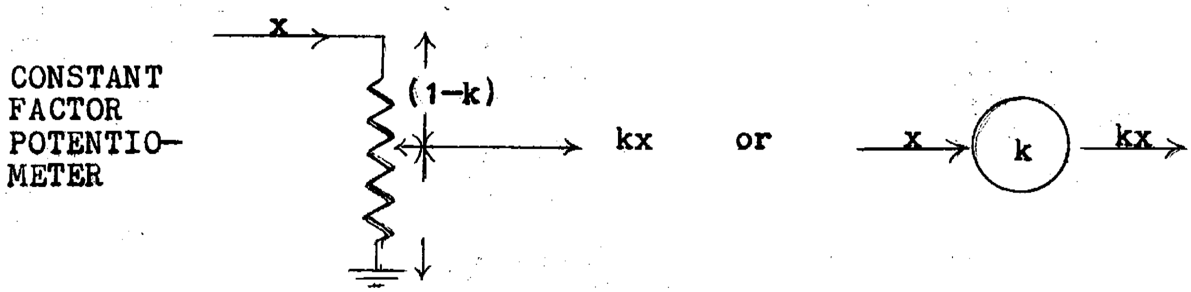
HIGH GAIN AMPLIFIER



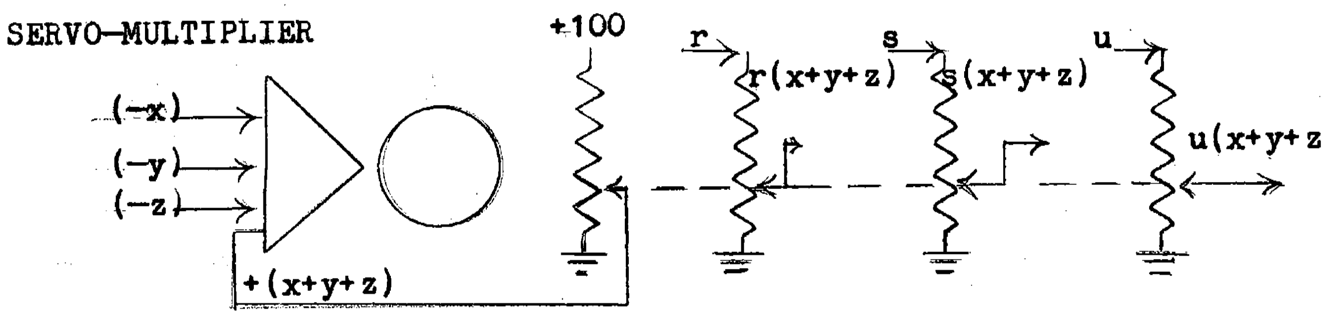
The following table summarizes the characteristics of the amplifiers:

Amplifiers	Input Gains	Integrate	Sum	Differentiate	High Gain
1– 7	.1, .25, 1, 1, 1	yes	yes	not conveniently	yes
8– 14	10, 4, 4, 1, 1, 1, 1	no	yes	no	no
15– 20	1, 1	yes	yes	no	yes
21– 23	1, 1, 1, 1	yes	yes	yes	yes

Since the inputs of the amplifiers permit multiplication by only a few different constants, factor potentiometers are necessary to provide arbitrary, fractional multiplication constants. The REAC has 24 constant factor potentiometers conveniently located on the panel above the amplifier jacks. A potentiometer is diagrammed as follows:



Potentiometers, similar to the constant factor potentiometers, are used for multiplication by a variable. However, instead of being positioned by hand, these potentiometers are positioned continuously by a servo-motor driven by a variable or the sum of variables. Each servo-motor drives three potentiometers and the following is the schematic notation for such a system:

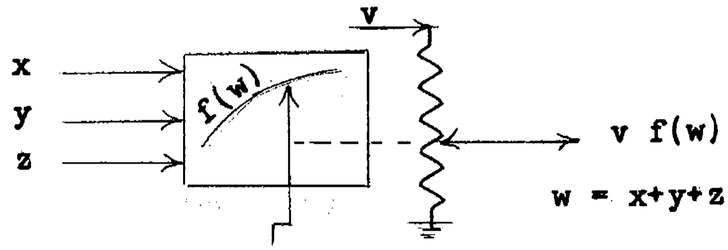


A slight modification of the above circuit is necessary if (x+y+z) changes sign. Sixteen such servo-multipliers are available in the RAND machine. An internal interconnection of three servo-multipliers permits vector resolution or summation; four such groupings are possible in the RAND REAC.

An arbitrary function of a variable or a sum of variables may be introduced by having an operator position an input table potentiometer according to a curve on a drum rotated by the variable, or sum of variables. Keuffel and Esser graph paper No. 359-11LG (10" x 15") is convenient for this application.

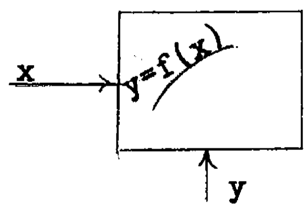
The input-tables are drawn as follows:

INPUT TABLE



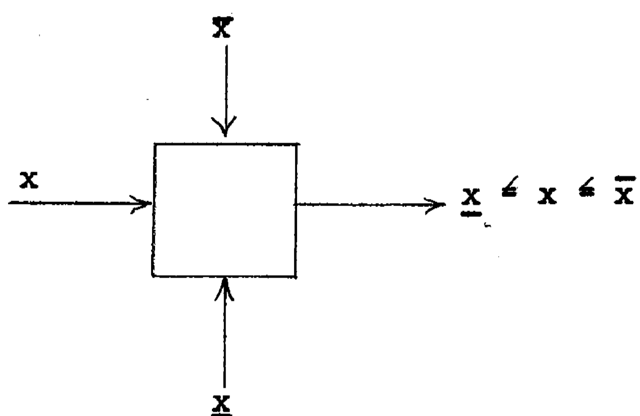
RAND has four such input-tables, plus an input-output table, an output table, and three Esterline-Angus recorders for plotting variables as a function of time. An output table is drawn as shown below (both variables may be the sum of as many as four variables):

OUTPUT TABLE



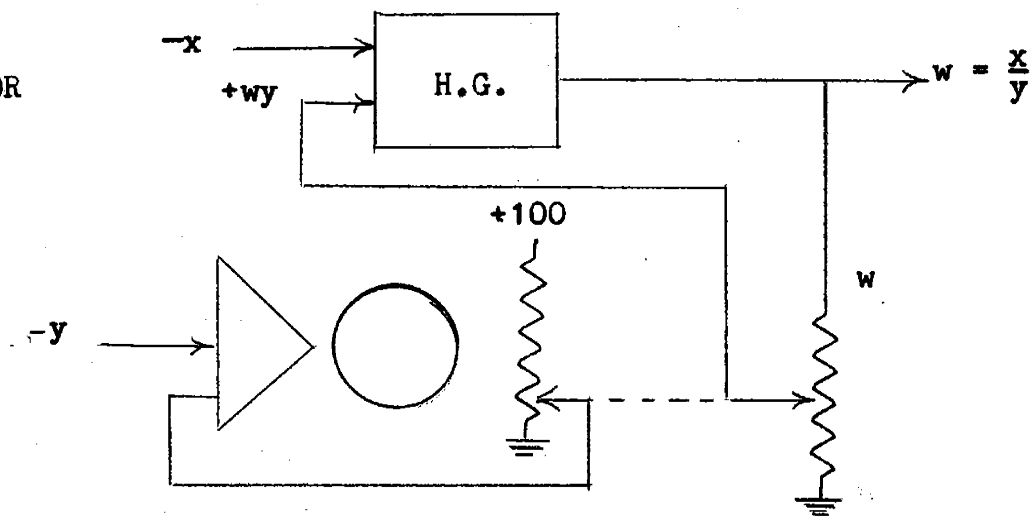
It is sometimes necessary to restrict a variable between two values and the REAC has four limiters to perform this task. The schematic representation for a limiter is as follows:

LIMITER



Division may be accomplished several ways, and the following method was selected for illustration because it demonstrates an application of a high-gain amplifier. Notice we want $w = \frac{x}{y}$ and the high-gain amplifier forces $(wy - x)$ towards zero. If y changes sign a more complicated circuit is necessary to prevent instability.

CIRCUIT FOR DIVISION

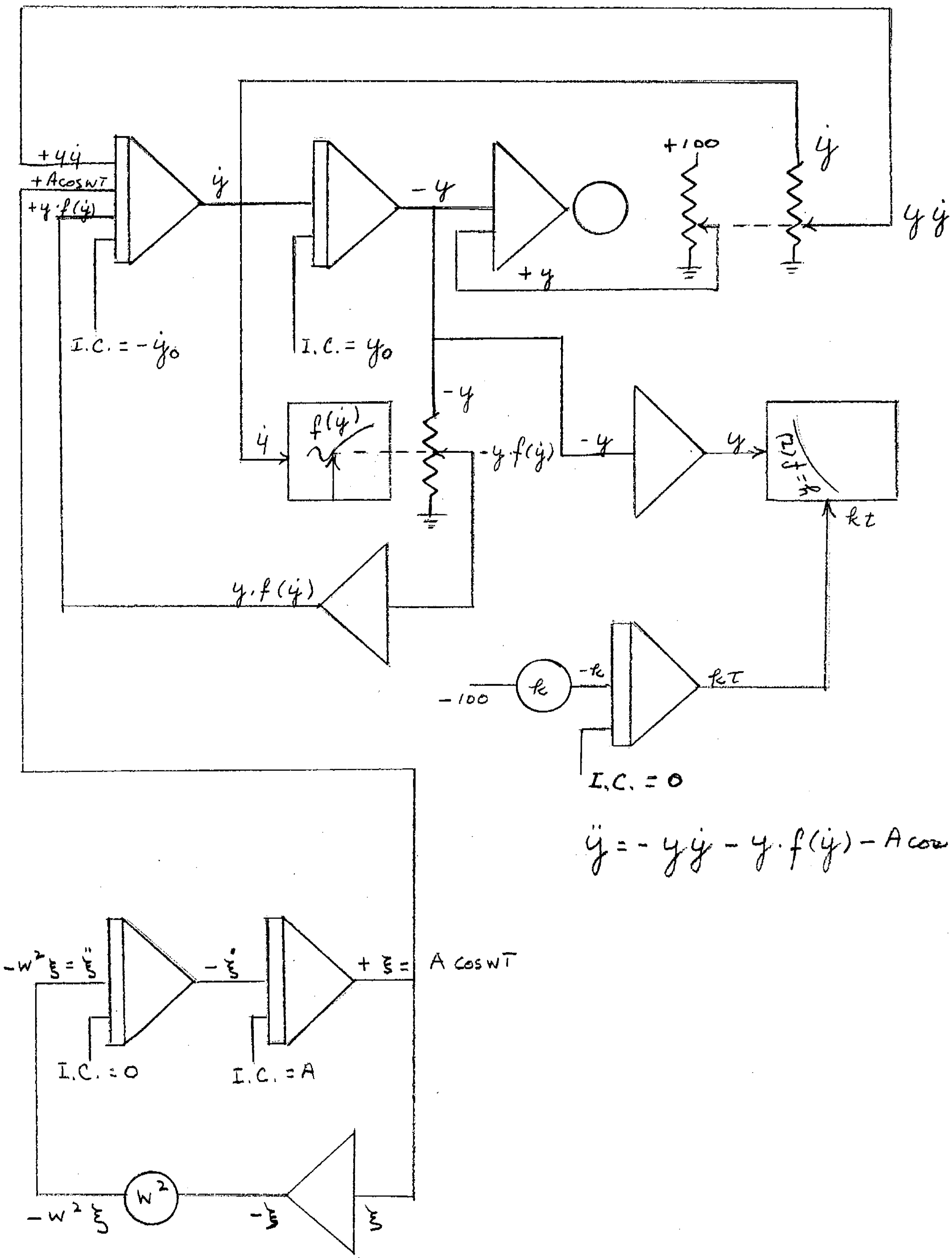


Example

As an example of how the components are interconnected for the solution of an equation, a schematic diagram is given below for the solution of $\ddot{y} = -y.\dot{y} - y.f(\dot{y}) - A\cos(wt)$. Notice we start the diagram assuming we have $-\ddot{y}$ available and operate on it until we develop the terms on the right side of the equation. The term $\xi = A\cos(wt)$ is developed by solving

$$\ddot{\xi} + w^2\xi = 0, \quad \xi_0 = A, \quad \dot{\xi}_0 = 0.$$

A tabular form is usually prepared, as well as a schematic, to aid in the plugging of a problem. The more advanced topics of scale factors, circuit stability, and utilization of equipment are of utmost importance, but cannot be covered in an elementary presentation.

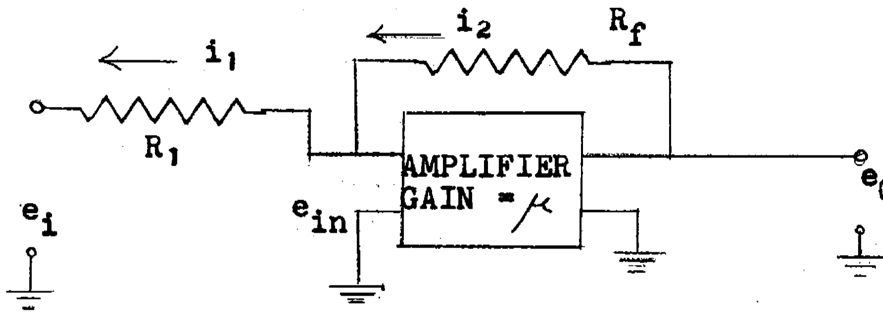


$$\ddot{y} = -y\dot{y} - y \cdot f(\dot{y}) - A \cos \omega T$$

APPENDIX

This appendix will show how the amplifiers accomplish their functions.

Summing Amplifier



$$i_1 = \frac{e_i - e_{in}}{R_1}$$

$$i_2 = \frac{e_{in} - e_O}{R_f}$$

$i_1 \approx i_2$, assuming no grid current

Since $u = \dfrac{e_O}{e_{in}}$ by definition

$$\frac{e_i + \frac{e_O}{u}}{R_1} = - \frac{\frac{e_O}{u} + e_O}{R_f} = \frac{e_O}{R_f}\left(1 + \frac{1}{u}\right)$$

or $$e_O = - e_i \frac{\frac{R_f}{R_i}}{1 + \frac{1}{u}\left(1 + \frac{R_f}{R_1}\right)} = e_i \frac{R_f}{R_1} \text{ for large } u.$$

Similarly,

$$i_1 = \frac{e_i}{R_1}\left[1 - \frac{1}{u}\frac{R_f}{R_1}\right] = \frac{e_i}{R_1}$$

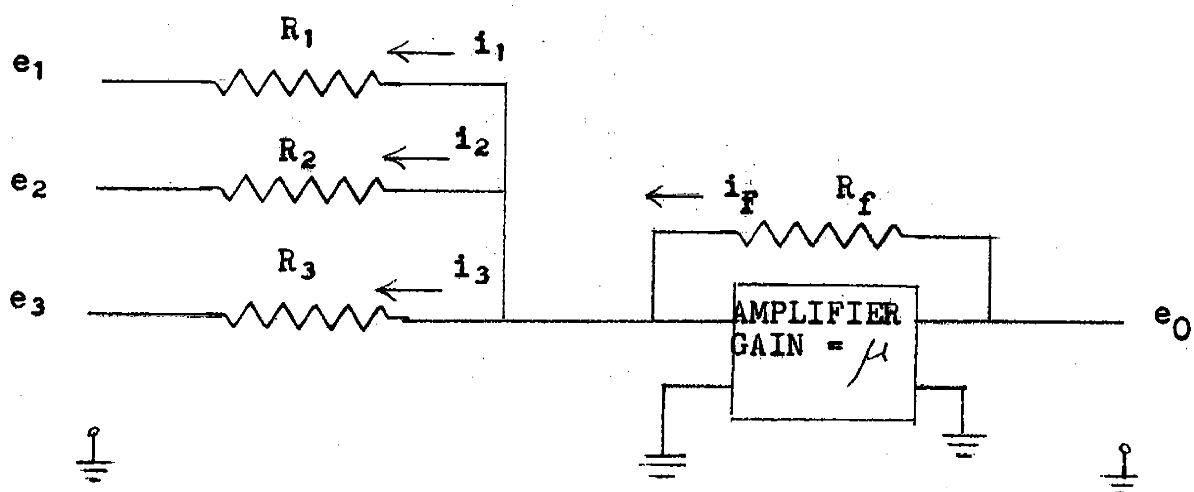
$$i_2 = - \frac{e_O}{R_f}\left(1 + \frac{1}{u}\right) = \frac{e_O}{R_f}$$

From this analysis, it follows that if there are several inputs, as shown below, the relationships are:

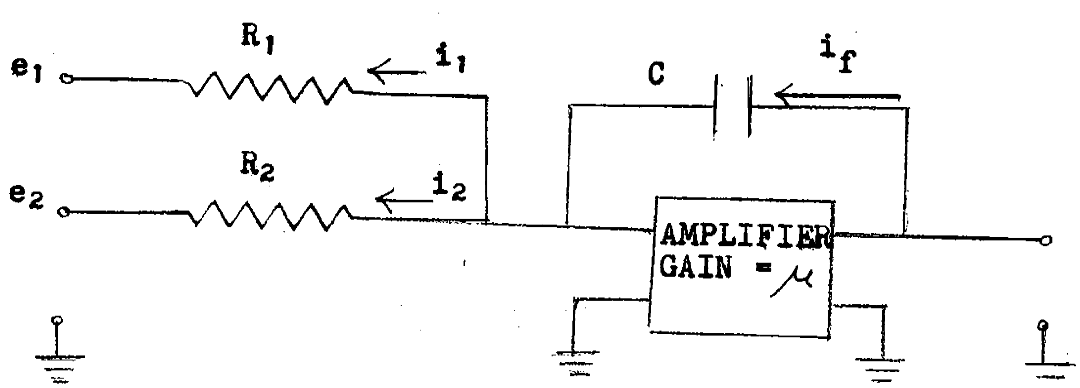
$$i_1 + i_2 + i_3 = i_f$$

$$\frac{e_1}{R_1} + \frac{e_2}{R_2} + \frac{e_3}{R_3} = \frac{e_0}{R_f}$$

$$e_0 = -\left(\frac{R_f}{R_1} e_1 + \frac{R_f}{R_2} e_2 + \frac{R_f}{R_3} e_3\right)$$



Integrating Amplifier



If the feedback resistor is replaced by a condensor the amplifier becomes an integrator. In this case,

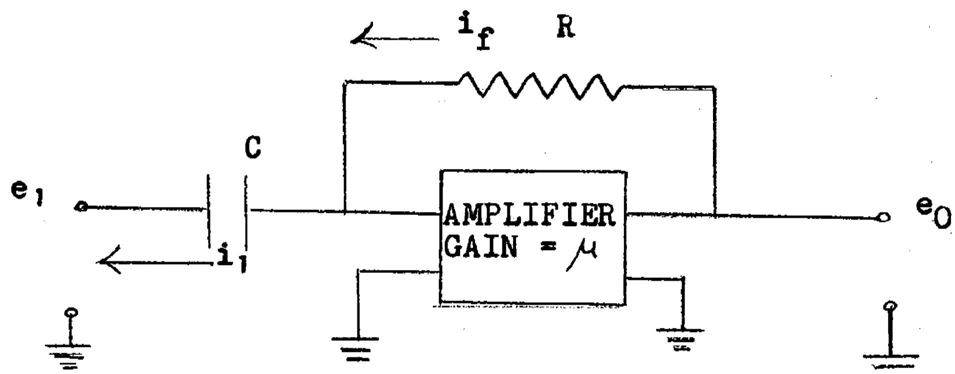
$$i_1 + i_2 = i_f$$

$$\frac{e_1}{R_1} + \frac{e_2}{R_2} = -\frac{Cde_0}{dt}$$

$$e_0 = -\left(\frac{1}{R_1C}\int_0^t e_1 dt + \frac{1}{R_2C}\int_0^t e_2 dt\right)$$

Differentiating Amplifier

In a differentiating amplifier a condensor is used as an input element.



$$i_1 = i_f$$

$$\frac{Cde_1}{dt} = -\frac{e_0}{R}$$

or $$e_0 = -RC\frac{de_1}{dt}$$

Sample Problem: The following problem will serve to test the reader's grasp of the fundamentals of preparing a problem for solution on the REAC:

$$\frac{dx}{dt} = ay + bx + c\frac{dy}{dt}$$

$$\frac{dy}{dt} = ay - bx - c\frac{dx}{dt}$$

$$x = y = A \text{ at } t = 0$$